# A2 OSCILLATIONS

## Oscillation/Vibration :-

Def. Repeated motion of an object about its mean position and between two extreme positions is called oscillation/vibration.

### Types :

(i) To and fro motion :-

Left and right side

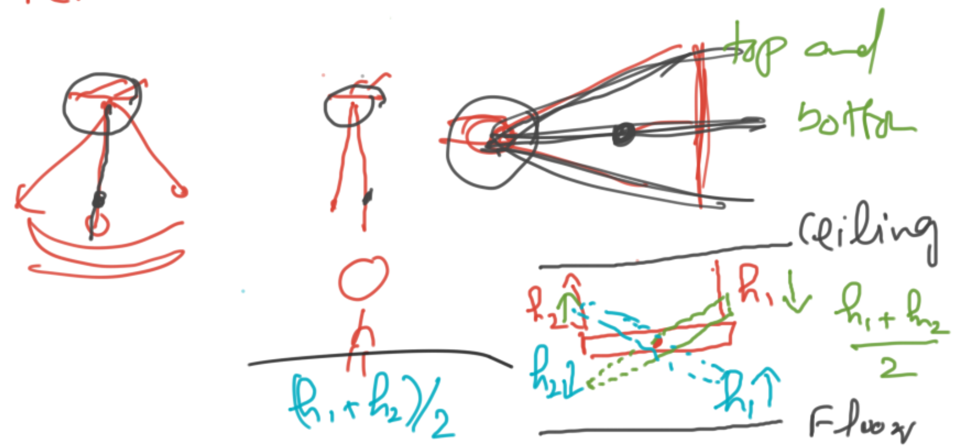
(ii) Back and forth :-

Increase and decrease of distance in front of you.

(iii) Top and bottom :-

upward and downward directions.

(iv) Torsional :- Motion about C.G. of object such that its distance from a fixed plane remain constant.



## Planes of oscillation:

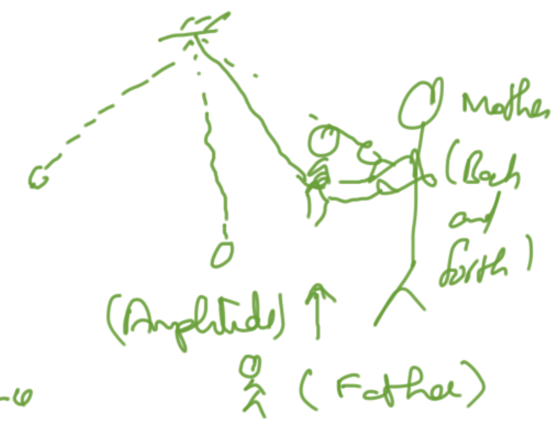
(i) Vertical plane :- Height or Gravitational potential energy of object changes during motion.

(ii) Horizontal plane:- Height or G.Ep of object remain constant during motion.

## Classification of oscillation

(a) Free oscillation:- Repeated motion of an object about its mean position and between two extreme position without variation in amplitude.

(b) Forced / Driving oscillation:- Repeated motion of an object about its mean position and between two extreme positions with a variation (increase / decrease) in amplitude

# Important terms

(i) Time period (T)

(ii) Frequency (f)

No. of complete oscillations per unit time.

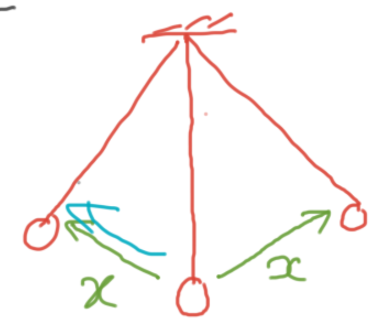
$f = \frac{n}{t}$ , n - no. of oscillations

For one oscillation , $n = 1$
$t = T$

$$\boxed{f = \frac{1}{T}}$$

(iii) Displacement :- ($x$)

Straight direction of an oscillating object from its mean position.




$x = x_0$
motion toward mean position : $x \downarrow$

Extreme

motion away from mean position
$x \uparrow$

(iv) Amplitude : ($x_0$)

Maximum displacement of an oscillating object.

P.S : Scalar

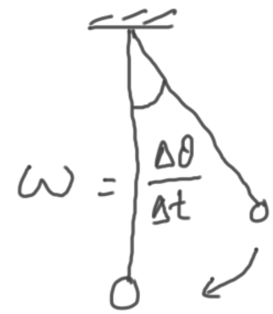
## (v) Periodic motion :-

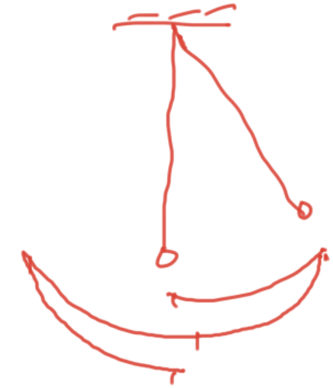
Motion which repeats itself in equal interval of time is periodic. ie

- Solar system (T = 24 h)
- Heart beat
- Simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$
- Mass attached to a spring $T = 2\pi\sqrt{\frac{m}{K}}$

## (vi) Angular frequency :-

Def: change of angle swept out by a vibrating object with its equilibrium position per unit time.



Symbol: $\omega$

Formula: $\omega = \frac{\Delta\theta}{\Delta t}$

For one complete oscillation

$\Delta\theta = 2\pi$ and $\Delta t = T$


$\omega = \frac{2\pi}{T} = 2\pi f$

Unit: rad $s^{-1}$

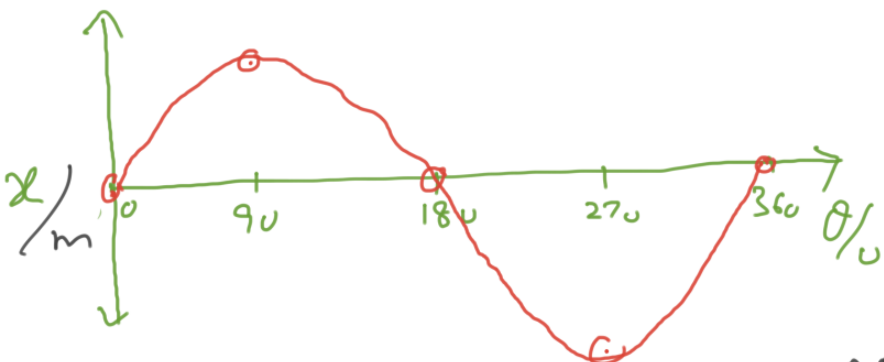
P.S: Scalar

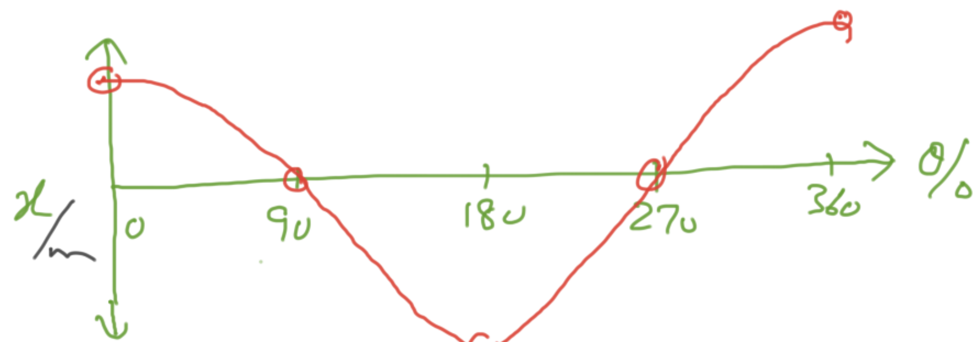
$x = \sin\theta$

| $\theta/°$ | 0 | 90 | 180 | 270 | 360 |
|---|---|---|---|---|---|
| $\sin(\theta/°)$ | 0 | 1 | 0 | −1 | 0 |

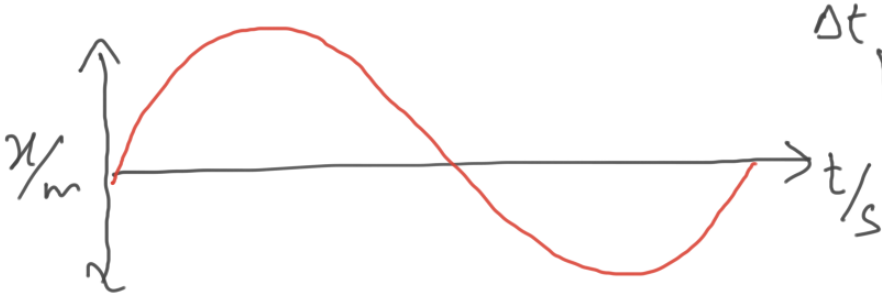
$x = \cos\theta$

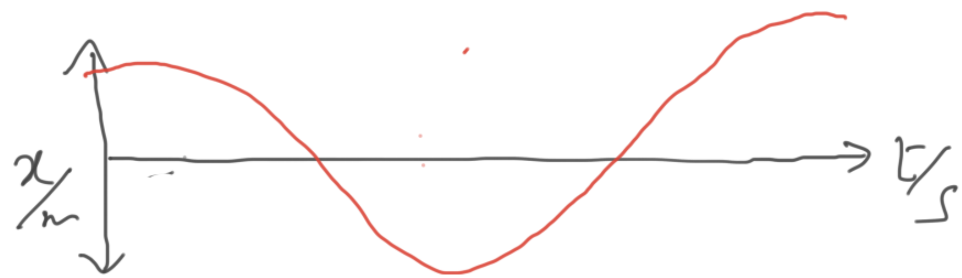
| $\theta/°$ | 0 | 90 | 180 | 270 | 360 |
|---|---|---|---|---|---|
| $\cos(\theta/°)$ | 1 | 0 | −1 | 0 | 1 |

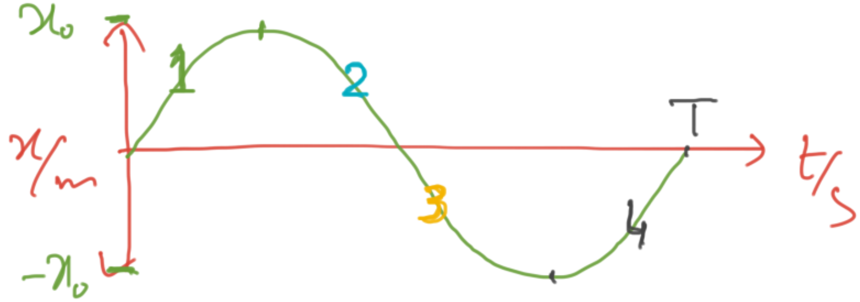


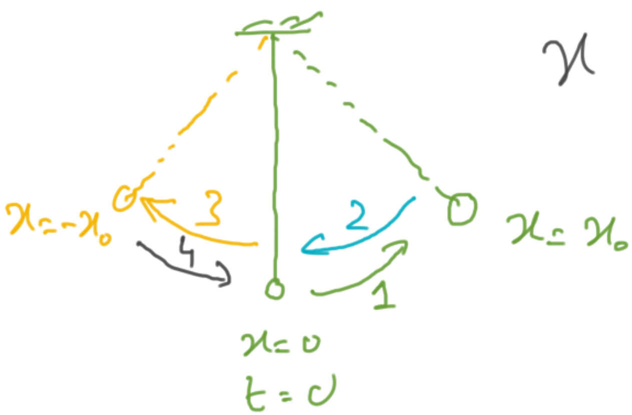

$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta\theta = \omega\Delta t$$



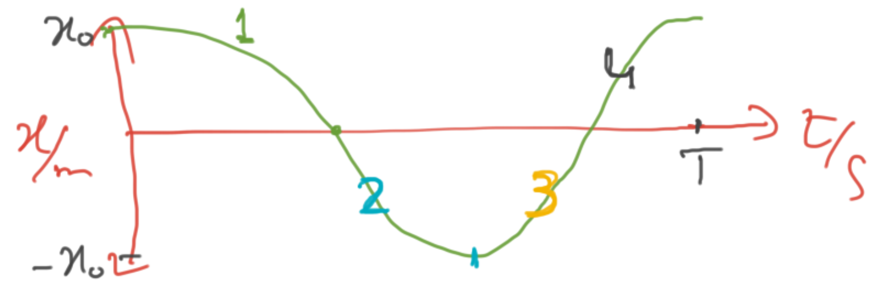



If stopwatch is started from mean position

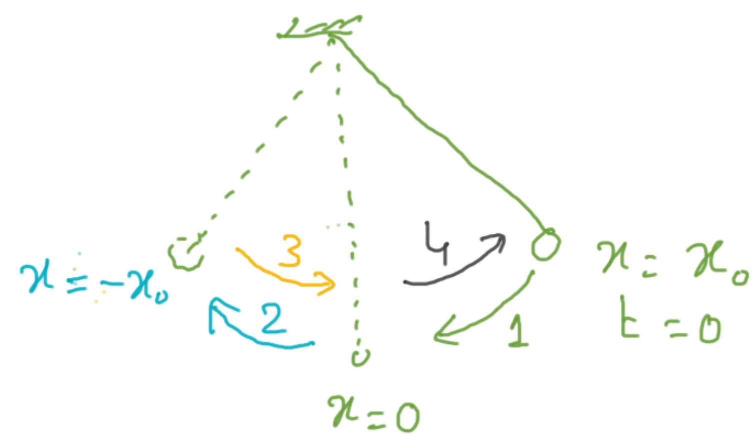



$$x = x_0 \sin\theta$$

$$x = x_0 \sin(\omega t)$$

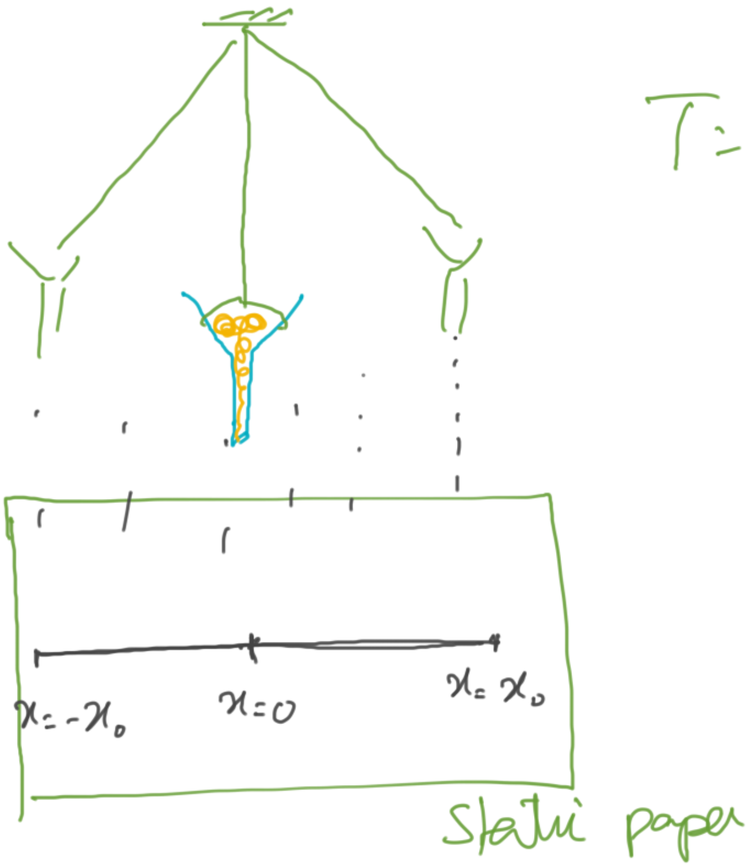



If stopwatch is started from extreme position

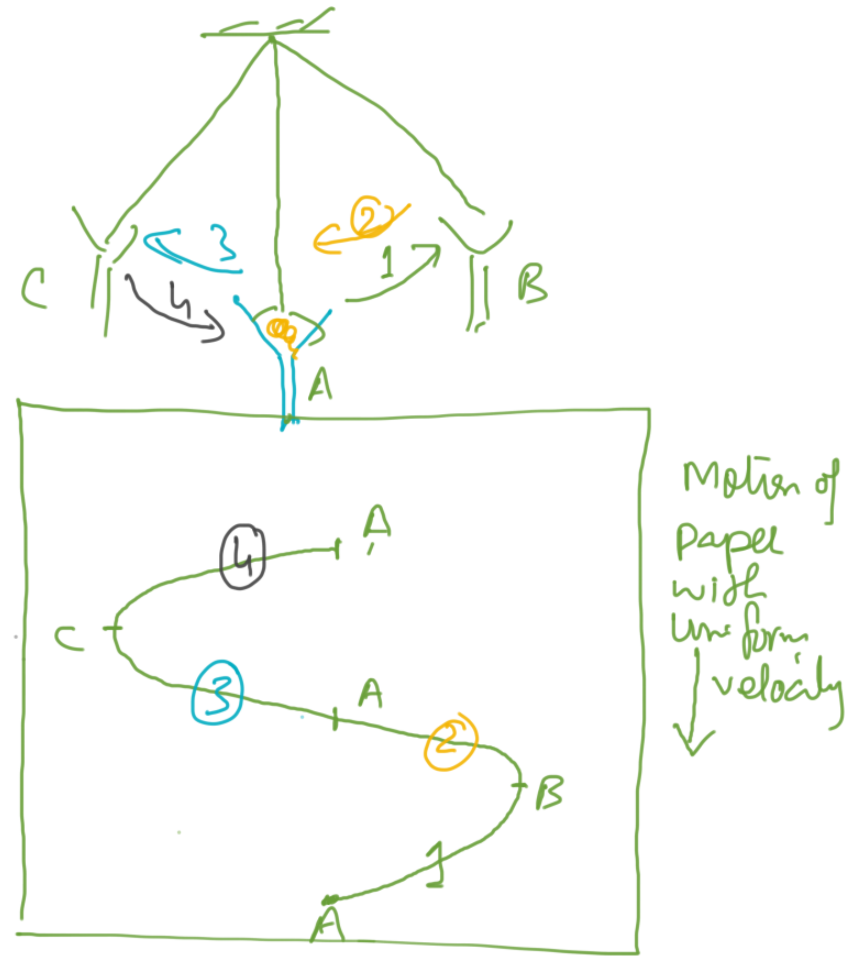



$$x = x_0 \cos\theta$$

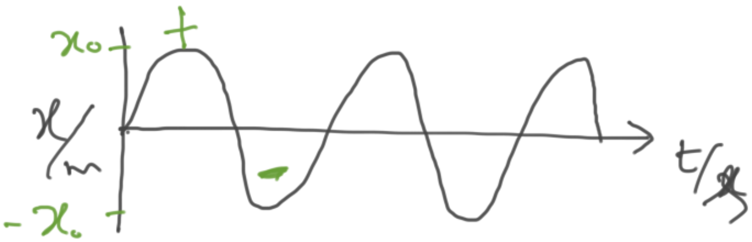
$$x = x_0 \cos(\omega t)$$



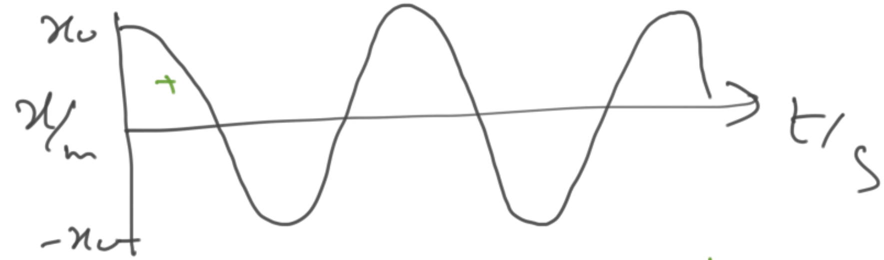

$$T = 2\pi\sqrt{\frac{l}{g}}$$



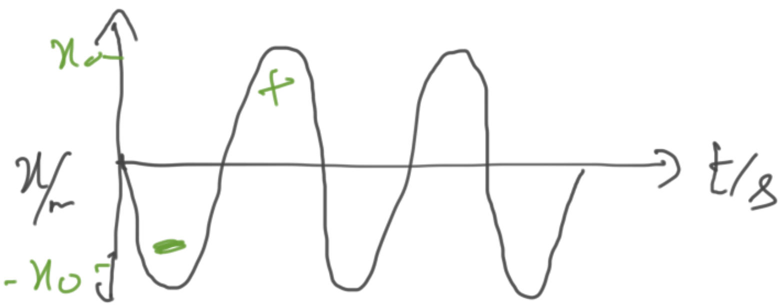

A vibrating object always transverse a wave form



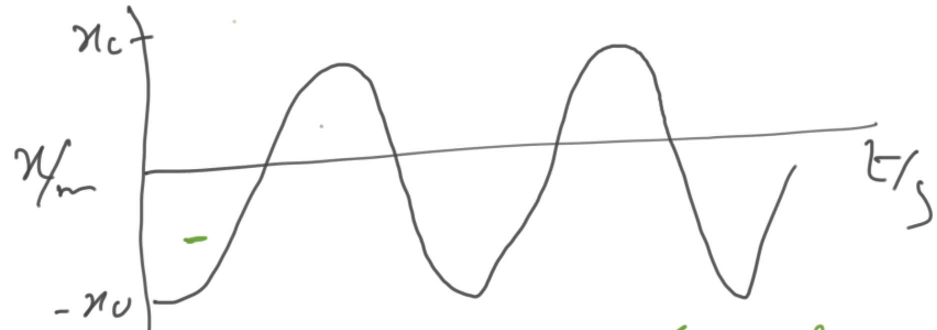

$x = x_0 \sin \omega t$




$x = x_0 \cos \omega t$




$x = -x_0 \sin \omega t$




$x = -x_0 \cos \omega t$

v.v. Imp

# Simple Harmonic motion (SHM)

* Periodic motion
* Magnitude of Acc. $\propto$ Displacement
* Direction of Acc. towards mean position or opposite to displacement

**Def.** Periodic motion in which acceleration is directly proportional to displacement and is directed towards mean position.

## Mathematical form:

$$a \propto -x$$

$$a = \omega^2(-x)$$

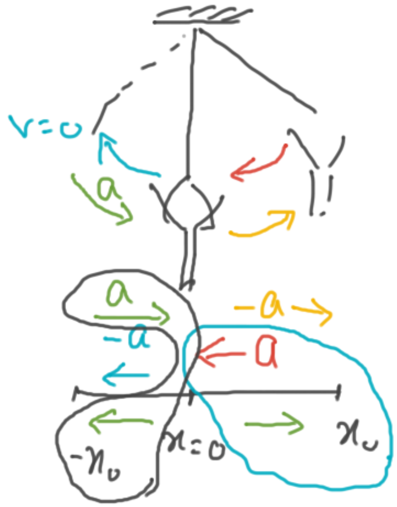
$$\boxed{a = -\omega^2 x}$$

Here $\omega = \frac{2\pi}{T} = 2\pi f$ = Constant

## Significance of -ve sign:

**Vectorial significance:** Both acceleration and displacement are in opposite directions.

**Conventional significance:** Acceleration is directed towards mean position




$a \propto -x$

# Kinematics' graphs

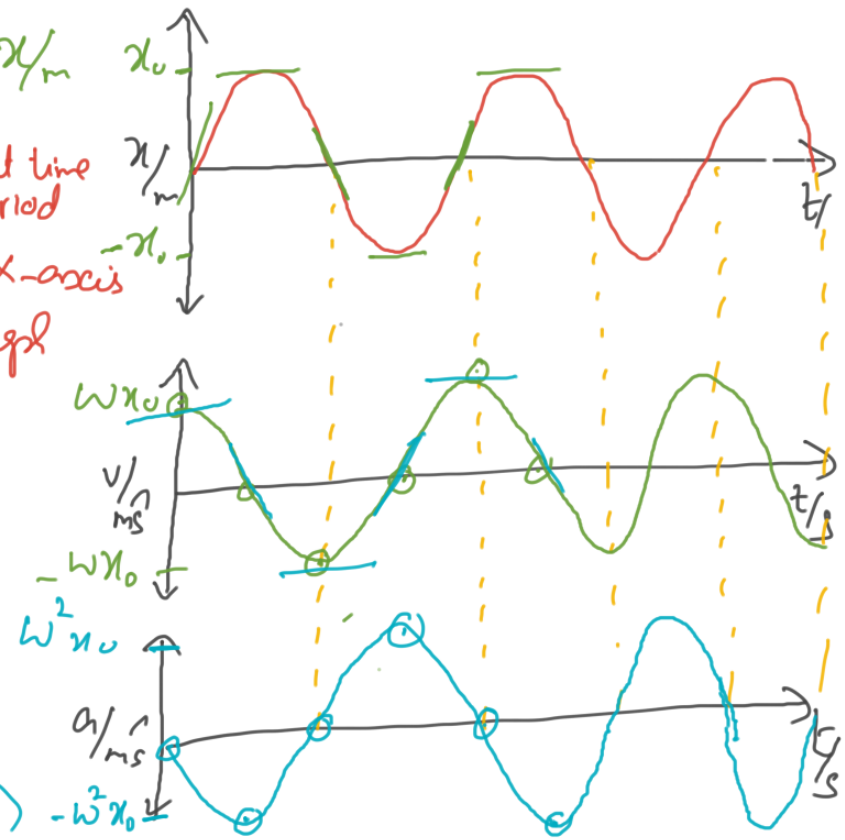
## (a) If stopwatch is started from mean position

### (i) Displacement time graph

The general eq. of x/m t/s graph is

$x = x_0 \sin \omega t$

Y-axis, Amplitude, Constant time period, Shape of graph, x-axis




### (ii) Velocity-time graph :-

V = Gradient of x/m - t/s graph

$V = \omega x_0 \cos \omega t$

Y-axis, Amplitude, trend of graph, x-axis

### (iii) Acceleration-time graph :

a = Gradient of $v/ms^{-1}$ - t/s graph

$a = \omega^2 x_0 (-\sin \omega t)$

Not in syllabus.

$$v = \frac{d}{dt}x = \frac{d}{dt}(x_0 \sin\omega t)$$

$$v = x_0 \frac{d}{dt} \sin\omega t$$

$$= x_0 \cos\omega t \left[\frac{d}{dt}\omega t\right]$$

$$= x_0 \cos\omega t \left\{\omega \frac{dt}{dt}\right\}$$

$$\boxed{v = \omega x_0 \cos\omega t}$$

$$a = \frac{d}{dt}v = \frac{d}{dt}(\omega x_0 \cos\omega t)$$

$$a = \omega x_0 \frac{d}{dt}(\cos\omega t)$$

$$a = \omega x_0 \left[-\sin\omega t \frac{d}{dt}(\omega t)\right]$$

$$a = -\omega x_0 \sin\omega t \left\{\omega \frac{dt}{dt}\right\}$$

$$\boxed{a = -\omega^2 x_0 \sin\omega t}$$

✓ $x = x_0 \sin\omega t$ — (1)

✓ $v = \omega x_0 \cos\omega t$ — (2)

$a = -\omega^2 x_0 \sin\omega t$ — (3)

Put (1) into (3)

$$\boxed{a = -\omega^2 x}$$

## (2) If stopwatch is started at extreme position

### (i) Displacement-time graph

$$\underbrace{x}_{\text{Y-axis}} = \underbrace{x_0}_{\text{Amplitude}} \underbrace{\cos \omega \overbrace{t}^{\text{x-axis}}}_{\text{Shape of graph}}$$

Constant T/s

### (ii) Velocity-time graph

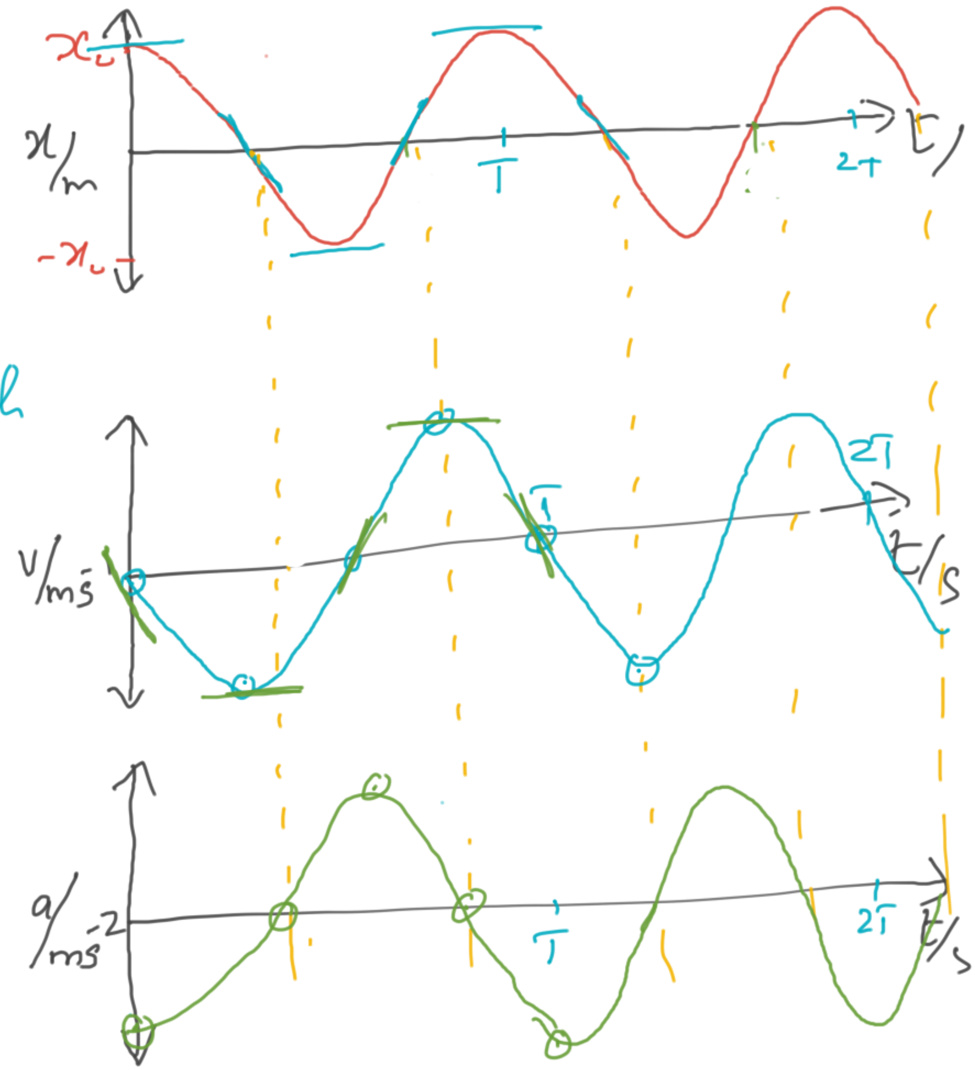
V = Gradient of $x/m - t/s$ graph

$$v = \underbrace{\omega x_0}_{\text{Amplitude}} \left(\underbrace{-\sin \omega t}_{\text{Shape of graph}}\right)$$

### (iii) Acceleration-time graph

a = Gradient of $v/ms^{-1} - t/s$ graph

$$a = \underbrace{\omega^2 x_0}_{\text{Amplitude}} \left(\underbrace{-\cos \omega t}_{\text{trend of graph}}\right)$$



# DYNAMICS' GRAPHS

## (i) Velocity - Displacement graph:

The given eq. of velocity displacement graph is

$$\boxed{V = \pm\, \omega \sqrt{x_0^2 - x^2}}$$

**At mean position:** $x = 0$

$$V = \pm\, \omega \sqrt{x_0^2 - (0)^2}$$

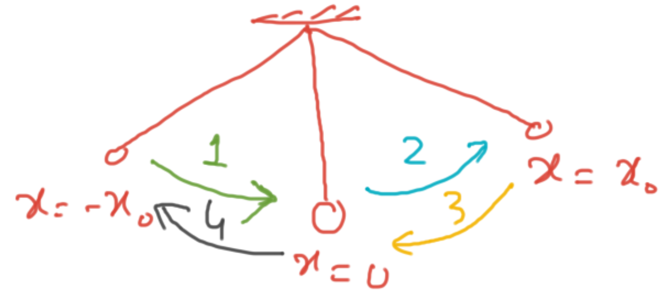
$$\boxed{V = \pm\, \omega x_0}$$

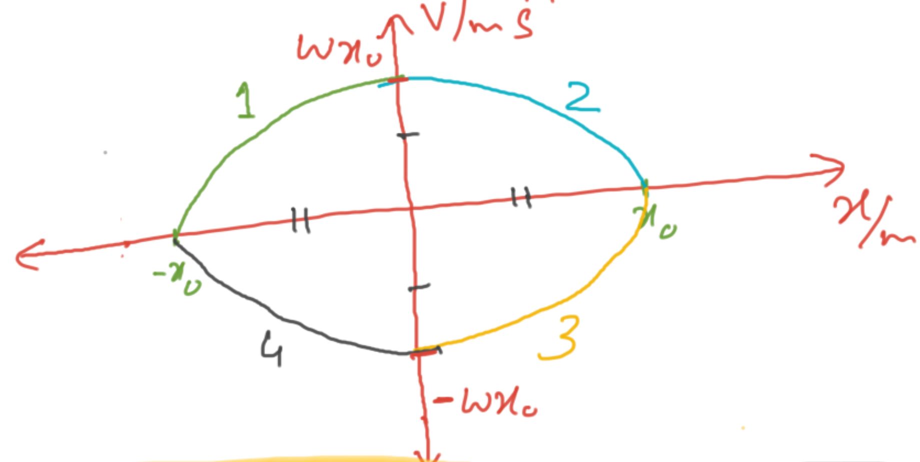
**At extreme position:** $x = \pm x_0$

$$V = \pm\, \omega \sqrt{x_0^2 - (\pm x_0)^2}$$

$$V = \pm\, \omega \sqrt{x_0^2 - x_0^2}$$

$$\boxed{V = 0}$$




Not in syllabus

$x = x_0 \sin\omega t \Rightarrow \sin\omega t = \dfrac{x}{x_0}$

$V = \omega x_0 \cos\omega t \Rightarrow \cos\omega t = \dfrac{V}{\omega x_0}$

Squaring and adding

$$\sin^2\omega t + \cos^2\omega t = \frac{x^2}{x_0^2} + \frac{V^2}{\omega^2 x_0^2} \Rightarrow 1 - \frac{x^2}{x_0^2} = \frac{V^2}{\omega^2 x_0^2}$$

$$V^2 = \omega^2 \cancel{x_0^2}\left[\frac{x_0^2 - x^2}{\cancel{x_0^2}}\right] \Rightarrow V = \pm\, \omega \sqrt{x_0^2 - x^2}$$

(ii) Acceleration - displacement graph :

$x = x_0 \sin\omega t$ —— (1)

$a = -\omega^2 x_0 \sin\omega t$ —— (2)

$x = x_0 \cos\omega t$ —— (1)

$a = -\omega^2 x_0 \cos\omega t$ —— (2)

Put eq. (1) into eq. (2)

$$\boxed{a = -\omega^2 x}$$

At mean position, $\boxed{x = 0}$

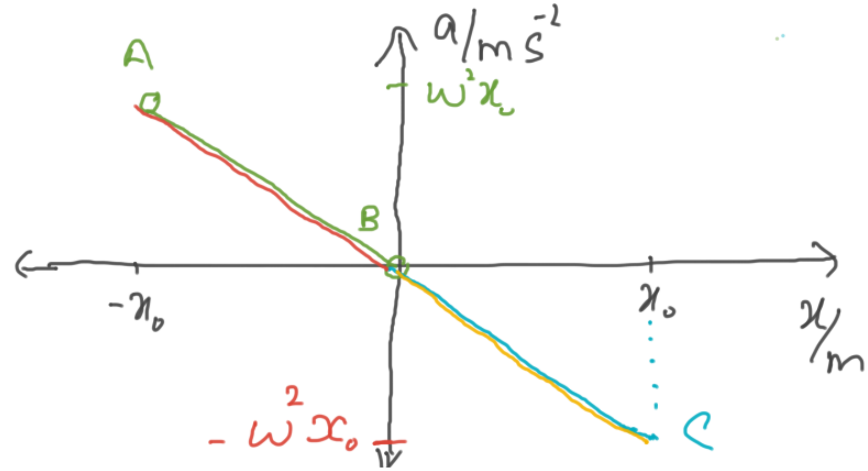
$a = -\omega^2(0) \Rightarrow \boxed{a = 0}$

At extreme position.

If $\boxed{x = +x_0} \Rightarrow \boxed{a = -\omega^2 x_0}$

If $\boxed{x = -x_0} \Rightarrow a = -\omega^2(-x_0)$

$$\boxed{a = \omega^2 x_0}$$

# ENERGY- DISPLACEMENT GRAPHS

## (i) Kinetic energy displacement graph:

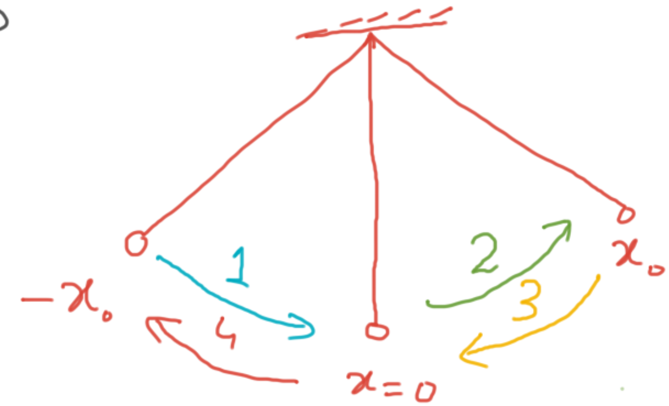
$$E_K = \frac{1}{2} m v^2$$

But $v = \pm \omega \sqrt{x_0^2 - x^2}$

$$E_K = \frac{1}{2} m \left(\pm \omega \sqrt{x_0^2 - x^2}\right)^2$$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

Above equation defines kinetic energy at any position ($x$).

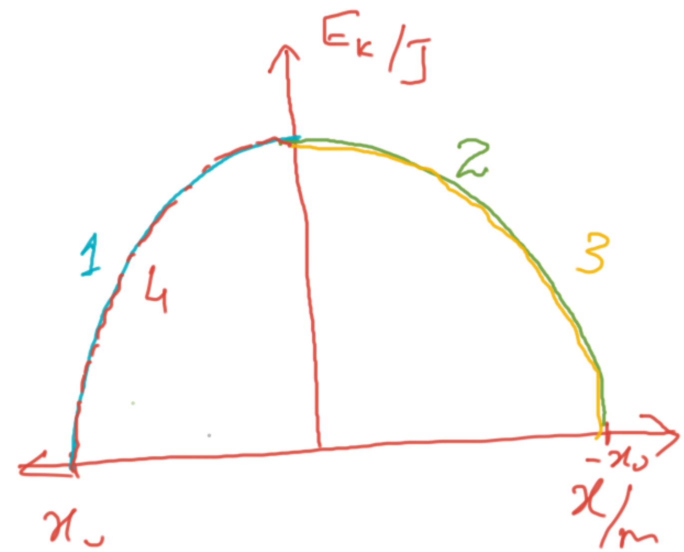



At mean position, $x = 0$

$$E_K = \frac{1}{2} m \omega^2 (x_0^2 - 0^2) \Rightarrow \boxed{E_K = \frac{1}{2} m \omega^2 x_0^2}$$

At extreme position, $x = \pm x_0$

$$E_K = \frac{1}{2} m \omega^2 \left[x_0^2 - (\pm x_0)^2\right] \Rightarrow \boxed{E_K = 0}$$



## (ii) Total energy - displacement graph

At any position $(x)$

$$E_T = E_K + E_p$$

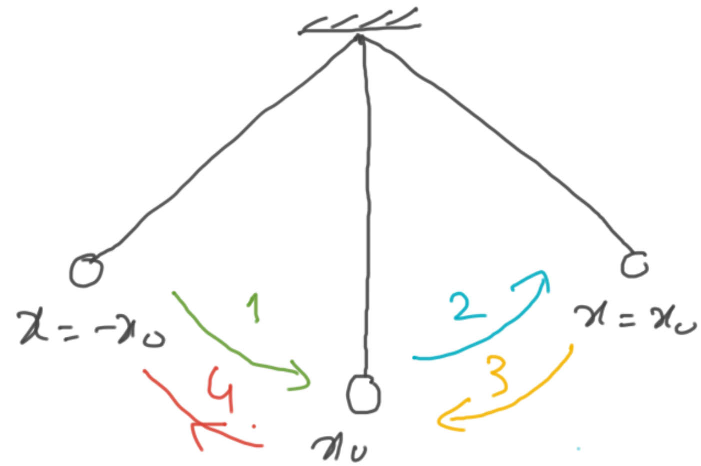
$$E_T = \frac{1}{2}m\omega^2(x_0^2 - x^2) + E_p$$

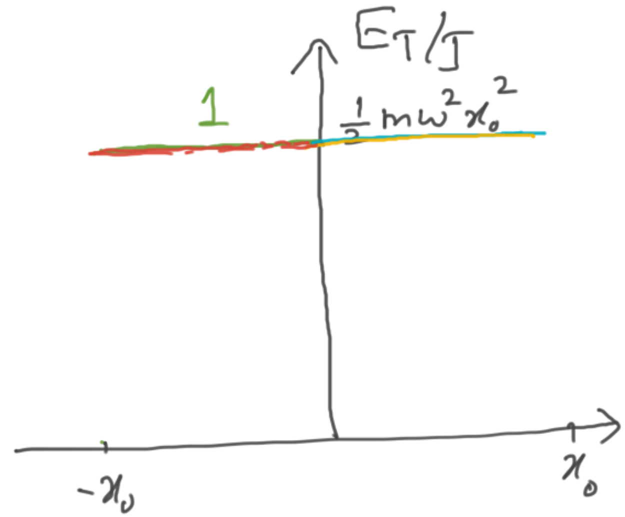
At mean position, $x = 0$, $E_p \approx 0$

$$E_T = \frac{1}{2}m\omega^2\left[x_0^2 - (0)^2\right] + 0$$

$$\boxed{E_T = \frac{1}{2}m\omega^2 x_0^2}$$

Since total energy is independant of displacement $(x)$, so it remain constant at all positions.



# Potential energy - displacement graph:

$$E_T = E_K + E_P$$

$$\frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2) + E_P$$

$$\cancel{\frac{1}{2}m\omega^2 x_0^2} = \cancel{\frac{1}{2}m\omega^2 x_0^2} - \frac{1}{2}m\omega^2 x^2 + E_P$$

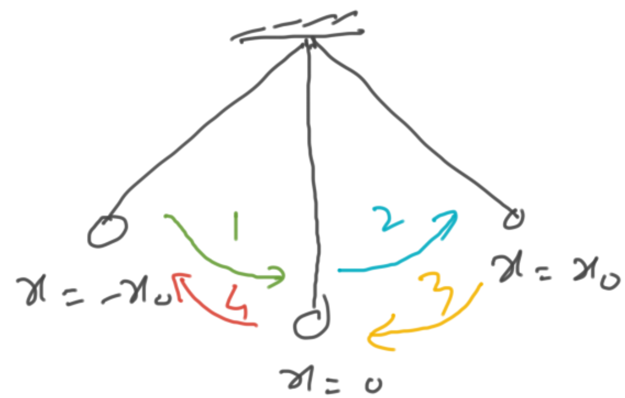
$$\boxed{E_P = \frac{1}{2}m\omega^2 x^2}$$

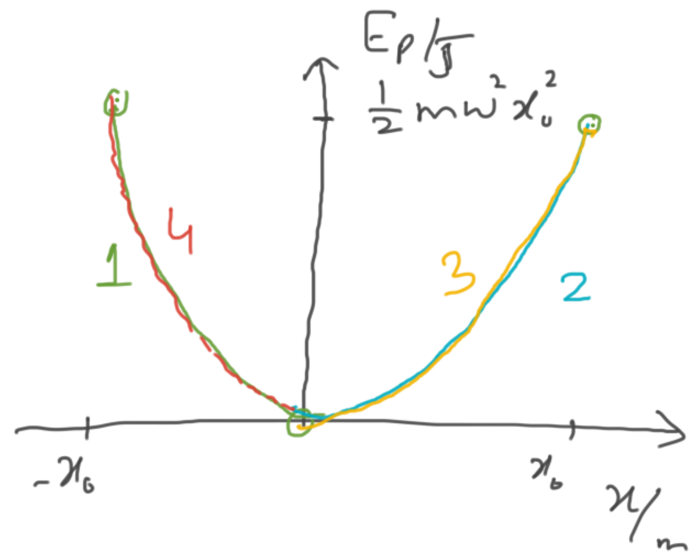
At mean position, $x = 0$, $E_P = 0$

At extreme position, $x = \pm x_0$

$$E_P = \frac{1}{2}m\omega^2(\pm x_0)^2$$

$$\boxed{E_P = \frac{1}{2}m\omega^2 x_0^2}$$



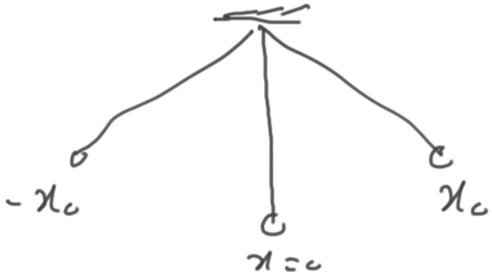



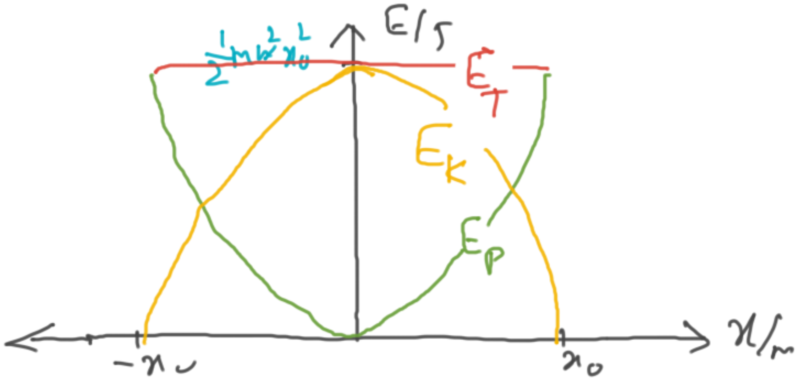
## Summary:

$$E_K = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

$$E_P = \frac{1}{2} m\omega^2 x^2$$

$$E_T = \frac{1}{2} m\omega^2 x_0^2$$




## Energy - time graphs :

**Case 1: If stopwatch is started from mean position**

### (a) Kinetic energy-time graph

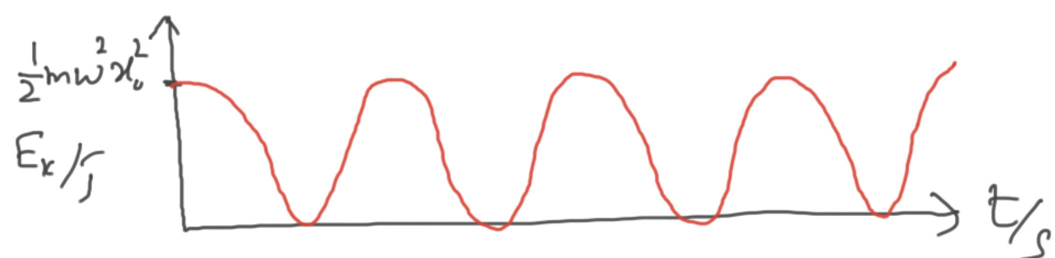
$$E_K = \frac{1}{2} m v^2$$

But $v = \omega x_0 \cos\omega t$

$$E_K = \frac{1}{2} m (\omega x_0 \cos\omega t)^2$$

$$\underset{\text{Y-axis}}{E_K} = \underbrace{\frac{1}{2} m\omega^2 x_0^2}_{\text{Amplitude}} \underbrace{\cos^2 \omega t}_{\text{Shape of graph}}$$

$\omega t \rightarrow$ x-axis



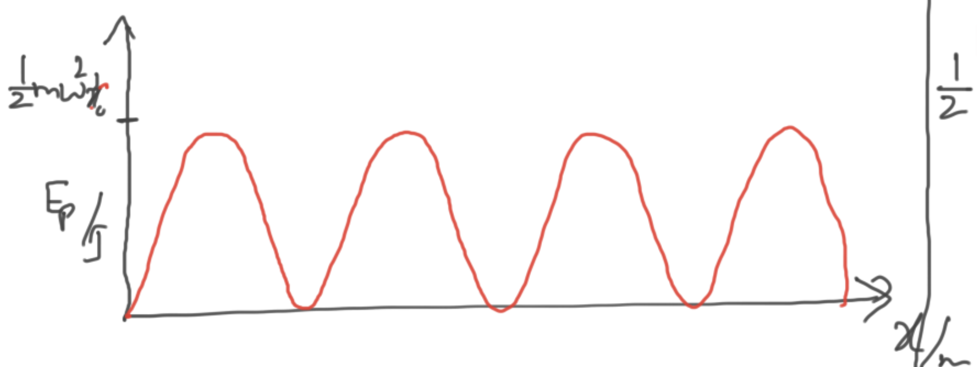
## Potential energy-time graph

$$E_p = \frac{1}{2} m\omega^2 x^2$$

But $x = x_0 \sin\omega t$

$$E_p = \frac{1}{2} m\omega^2 (x_0 \sin\omega t)^2$$

$$E_p = \underbrace{\frac{1}{2} m\omega^2 x_0^2}_{\text{Amplitude}} \; \underbrace{\sin^2\omega t}_{\text{Shape of graph}}$$




## Total energy-time graph

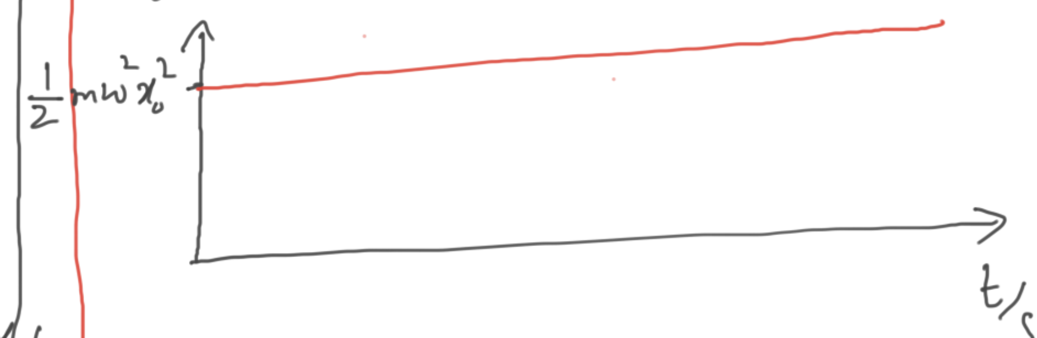
$$E_T = E_K + E_p$$

$$= \frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t + \frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t$$

$$= \frac{1}{2} m\omega^2 x_0^2 (\cos^2\omega t + \sin^2\omega t)$$

$$\boxed{E_T = \frac{1}{2} m\omega^2 x_0^2}$$

Since total energy is independent of time and remain constant.



## Case 2: If stopwatch is started from extreme position:-

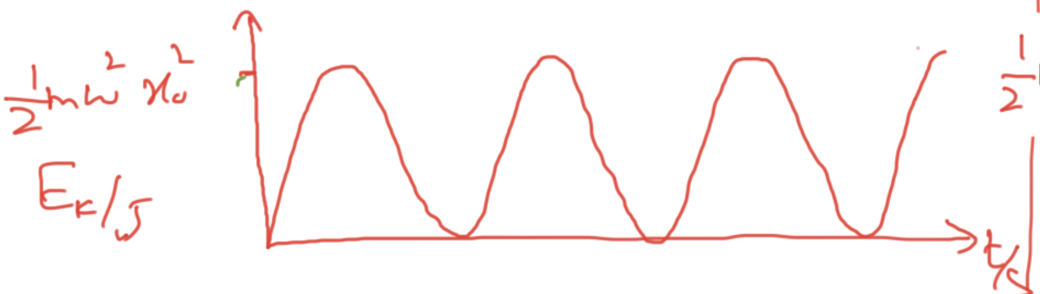
### (i) Kinetic energy-time graph

$$E_K = \frac{1}{2} m v^2$$

But $v = \omega x_0 (-\sin \omega t)$

$$E_K = \frac{1}{2} m (-\omega x_0 \sin \omega t)^2$$

$$= \underbrace{\frac{1}{2} m \omega^2 x_0^2}_{\text{Amplitude}} \; \underbrace{\sin^2 \omega t}_{\text{Shape of graph}}$$

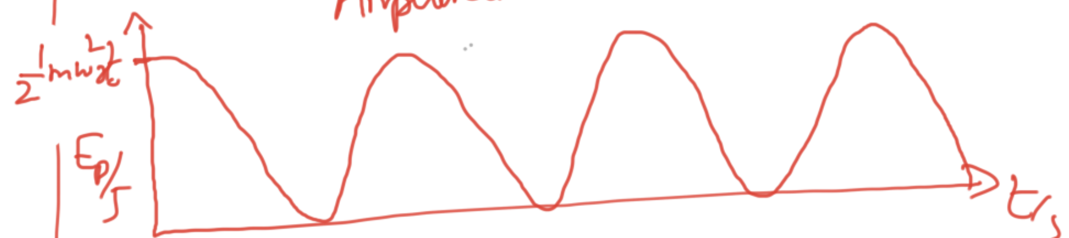



### (ii) Potential energy-time graph

$E_p = \frac{1}{2} m \omega^2 x^2$ But $x = x_0 \cos \omega t$

$$E_p = \frac{1}{2} m \omega^2 (x_0 \cos \omega t)^2$$

$$E_p = \underbrace{\frac{1}{2} m \omega^2 x_0^2}_{\text{Amplitude}} \; \underbrace{\cos^2 \omega t}_{\text{trend of graph}}$$




### (iii) Total energy-time graph

$$E_T = E_K + E_p = \frac{1}{2} m \omega^2 x_0^2$$